

**Linear Operators in Hilbert Space and
Their Applications**

BY

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ABSTRACT

In this study we explain linear system which is one of basic importance in functional analysis , then linear function and linear operators .

We study inner product space which will be needed in Hilbert space . The study discusses some theorems and definitions related to the field . The linear differential operator is also explained in details .

The study concludes that the linear differential operators can be uses to solve ordinary differential equations that satisfy some boundary conditions .

ملخص البحث

في هذا البحث تناولنا النظام الخطى لأهميته كأساس في التحليل الدالى ثم الدوال الخطية والمؤثرات الخطية .

ودرسنا فضاء الضرب الداخلى كتمهيد لدراسة فضاء هلبيرت وناقشت الدراسة بعض التعاريف

والنظريات في هذا المجال وركزت الدراسة على المؤثر التفاضلى الخطى .

وخلصت الدراسة في استخدام المؤثر التفاضلى الخطى لحل المعادلات التفاضلية العادية ذات شروط

حدية معينة .

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